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$\therefore$ The required harmonical progression is

$$1/(p^2-q^2-2pq)^2,\ 1/(p^2+q^2)^2,\ 1/(p^2-q^2+2pq)^2,$$

in which p and q are any integers, $p>q$.

Put $p=2$ and $q=1$. We then have $1/1^2$, $1/5^2$, $1/7^2$.

Put $p=3$ and $q=2$. We then have $1/7^2$, $1/13^2$, $1/17^2$.

II. Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

If a^2, b^2 are two numbers, then will a^2, $2a^2b^2/(a^2+b^2)$, b^2 be in harmonical progression. Let $a=m-n$, $b=m+n$; therefore, $(m-n)^2$, $(m^2-n^2)^2/(m^2+n^2)$, $(m+n)^2$ will be the required numbers, if $m^2+n^2=\square$.

Let $m=p^2-q^2$, $n=2pq$.

$\therefore (q^2+2pq-p^2)^2$, $(p^4-6p^2q^2+q^4)^2/(p^2+q^2)^2$, $(p^2+2pq-q^2)^2$ are the numbers required.

Let $p=2$, $q=1$. $\therefore 1$, $\frac{49}{25}$, 49 are the numbers.

Let $a=m$, $b=7m$. Then m^2, $49m^2/25$, $49m^2$ are the numbers required.

Let $m=5n$. $\therefore 25n^2$, $49n^2$, $1225n^2$ are the required numbers. This gives a series of whole numbers by giving n integral values.

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.; A. H. BELL, Hillsboro, Ill.; and CHAS. C. CROSS, Meredithville, Va.

As the reciprocal of numbers in arithmetical progression are in harmonical progression, we may solve the question by obtaining three squares in arithmetical progression.

Let $x^2-2xy+y^2$, x^2+y^2, and $x^2+2xy+y^2$, be the three squares, and it only remains to make $x^2+y^2=\square$. This is so when $x=p^2-q^2$ and $y=2pq$; then $x^2+y^2=(p^2+q^2)^2$ in which p and q may be any unequal numbers.

Take $p=2$, and $q=1$, and the squares are 1, 25, and 49, and the reciprocals are 1, $\frac{1}{25}$, and $\frac{1}{49}$, or if integrals are desired, 1225, 49, and 25.

Also solved by *O. S. WESTCOTT, SYLVESTER ROBINS, ALOIS F. KOVARIK, J. W. YOUNG*, and the *PROPOSER*.

79. Proposed by EDMUND FISH, Hillsboro, Ill.

Find an integral right triangle in which the bisector of one of the acute angles is also integral.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

In any right triangle, let a=altitude, b=base, c=hypotenuse, c_1=bisector of angle opposite base, c_2=bisector of angle opposite altitude.

Put $a=tpq$, $b=t(p^2-q^2)/2$, and $c=t(p^2+q^2)/2$; t, p, and q being any integers, $p>q$.

Then from solution I, problem 43, page 95, Vol. IV, No. 3, THE AMERICAN MATHEMATICAL MONTHLY, we find

$$c_1=\frac{tpq\sqrt{[2(p^2+q^2)]}}{p+q}, \text{ and } c_2=\frac{t(p^2-q^2)\sqrt{(p^2+q^2)}}{2p}.$$

c_2 will be rational and integral when $p^2+q^2=\square$, and $t=2p$. The general values for p and q, in this case, are $2mn$ and m^2-n^2, $m>n$, $p>q$.

Put $m=2$ and $n=1$. Then $p=4$, $q=3$, and $t=8$.

Whence, $a=96$, $b=28$, $c=100$, and $c_2=35$.

Put $m=3$, and $n=2$. Then $p=12$, $q=5$, and $t=24$.

Whence, $a=1440$, $b=1428$, $c=2028$, and $c_2=1547$.

c_1 will be rational and integral when $p^2+q^2=2\times\square$, and $t=\frac{1}{2}(p+q)$.

The general values for p and q, in this case, are m^2-n^2+2mn and m^2-n^2 the difference between $2mn$, $m>n$ and $p>q$. The numerical results are the same as those obtained in case of c_2, except that a and b have interchanged values.

Of course, only one of the bisectors can be rational in any one triangle.

II. Solution by A. H. BELL, Hillsboro, Ill.

Let the hypotenuse$=h$, and the two legs$=a$ and $2c$, bisector$=b$, and segments $c+s$ and $c-s$.

Then $ah=(c+s)(c-s)+b^2$. $\therefore b^2=ah-(c^2-s^2)$ .... (1).

$a(c+s)=h(c-s)$. $\therefore s=c(h-a)/(h+a)$......(2).

Substituting the general value of the sides of a right triangle in (1) and (2), m^2+n^2, m^2-n^2, $2mn$; $s=n^3/m$, $s^2=n^6/m^2$.

$b^2=(m^2+n^2)(m^2-n^2)^2/m^2=\square$, if m and n are taken for the legs of any integral right triangle. When all the parts are multiplied by m, we obtain integral values, and they are $m(m^2+n^2)$, $m(m^2-n^2)$, and $2m^2n$ for the sides; $\sqrt{[(m^2+n^2)]}(m^2-n^2)$=bisector ; $n(m^2\pm n^2)$=segments.

In the right triangle 5, 4, 3, take m and $n=4$ and 3; then the required right triangle is 100, 28, 96; bisector=35, segments=75 and 21. In the right triangle 13, 12, 5, m and $n=12$ and 5; then the required sides=2028, 1428 and 1440, bisector=1547, segments=845 and 595.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let ABC be a right triangle, right angled at C, AD the bisector of $\angle A$.

Let $BC=a$, $AC=b$, $AB=c$, $AD=d$.

Then $\frac{1}{2}cd\sin\frac{1}{2}A+\frac{1}{2}bd\sin\frac{1}{2}A=\frac{1}{2}bc\sin A$. $cd+bd=2bc\cos\frac{1}{2}A$.

$\therefore d=2bc\cos\frac{1}{2}A/(b+c)$. Also $d=b\sec\frac{1}{2}A$.

$\therefore \sec\frac{1}{2}A=\sqrt{[2c/(b+c)]}$. $\therefore d=b\sqrt{[2c/(b+c)]}$.

Let $a=2pq$, $b=p^2-q^2$, $c=p^2+q^2$. $\therefore \sqrt{[2c/(b+c)]}=\sqrt{(p^2+q^2)}/p$.

Let $p=m^2-n^2$, $q=2mn$. $\therefore \sqrt{[2c/(b+c)]}=\sqrt{(p^2+q^2)}/p=(m^2+n^2)/(m^2-n^2)$.

Let $m=3r$, $n=r$. Then $p=8r^2$, $q=6r^2$. $\therefore a=96r^4$, $b=28r^4$, $c=100r^4$, $d=35r^4$, where r can have any integral value.

IV. Solution by O. S. WESTCOTT, A. M., Sc. D., Principal North Division High School, Chicago, Ill.

$BD : DC :: x^2+y^2 : x^2-y^2$, and $BD+DC$ or $BC : DC :: 2x^2 : x^2-y^2$.

$\therefore DC=2xy(x^2-y^2)/2x^2=[(x^2-y^2)/x]y$.
And $AD^2=(x^2-y^2)^2+y^2[(x^2-y^2)^2/x]y^2$.
$AD^2=(x^2+y^2)(x^2-y^2)^2/x^2$. $AD=[(x^2-y^2)/x]\surd(x^2+y^2)$.

Put $x=4$, $y=3$, and $AD=8\frac{3}{4}$. Increasing each dimension of triangle in quadruple ratio $AD=35$, $AC=28$, $BC=96$, $AB=100$, $DC=21$.

Also solved by ***CHAS. C. CROSS***, ***JOSIAH H. DRUMMOND***, and ***COOPEB D. SCHMITT***.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

128. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

At what time is the figure 7, on the face of a clock, midway between the hour and minute hands?

129. Proposed by J. W. DAPPERT, Civil Engineer and Surveyor, Taylorville, Ill.

"A Minion, agile, in stature small
Panting came to great Diana's Hall,
Bearing a marble globe upon his shoulders,
Measuring one inch in its diameters.
He rolled it to the northeast corner of the Hall,
Left touching the northern and eastern walls;
Then following came three demi-gods in white,
Each bearing a globe of lustrous metal bright;
One of iron, copper one, and one of silver;
And they placed them in the order given,
Touching each the other, and at the same time,
Touching each the side-walls, in a direct line,
The iron touching the marble, and its other side
Resting against the silver, in its glory and pride,—
All resting upon the oaken floor; and then
With heavy tread, and puff, and roar, Atlas came
Bearing a huge golden sphere, that filled the Hall,
Touching the four sides, floor and ceiling, and all
Radiant with beauty, resting against the silvery ball,
Making the globe's diameters in the rooms diagonal."

"Tell me, all ye who mathematics know:
What size the copper sphere, and oh!
How large the iron globe? How great
The golden globe, immaculate?
The silver sphere, how great? What size?
And if presented as a prize,
What value do you hold
Would be the sphere of gold?"

Found in an old volume entitled "Treatise on Surveying, by Abel Flint"—published in 1833, owned formerly by John E. Stockton, later by S. W. Baker, and now in my possession. Written in red ink upon a blank page.

⁂ Solutions of these problems should be sent to B. F. Finkel not later than May 10.